UNIT ⇒ IV :→ ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

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Chapter ⇒ 6

ELECTROMAGNETIC INDUCTION

MAGNETIC FLUX

'Magnetic flux linked with a surface in a magnetic field is defined as the number of Magnetic field lines crossing the surface normally.'

⇒ It is a Scalar quantity, denoted by 'ϕ'.

⇒ It is measured as :→

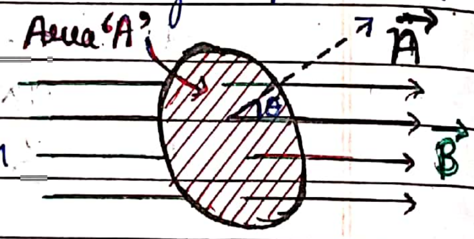
$$\boxed{\phi = \vec{B}\cdot\vec{A}} \quad \text{or} \quad \boxed{\phi = BA\cos\theta}$$

where,

$\vec{B}$ ⇒ Magnetic field vector

$\vec{A}$ ⇒ area vector and θ ⇒ angle b/w $\vec{B}$ & $\vec{A}$

Here, the direction of $\vec{A}$ is the direction of the outward drawn normal to the surface.



S.I. Unit ⇒ $\boxed{Tm^2}$ or $\boxed{\frac{\text{weber}}{m^2} \times m^2 = \text{WEBER (Wb)}}$

or $\boxed{\frac{N}{Am} \times m^2 = NmA^{-1}}$

'One WEBER is the flux produced when a uniform magnetic field of 1 Tesla acts normally over an area of $1\ m^2$.'

1 Weber = 1 Tesla × $1 m^2$ or $\boxed{1 Wb = 1 Tm^2}$

CGS Unit ⇒ $\boxed{G\,cm^2}$ = $\boxed{\text{MAXWELL } (Mx)}$

$\boxed{1 Mx = 1 G\,cm^2}$ G ⇒ Gauss = 10^{-4} T

Relation b/w Weber and maxwell :→

$$1 \text{ Weber} = 1T \times 1m^2$$

$$1 \text{ Weber} = 10^4 G \times 10^4 cm^2 = 10^8 G cm^2$$

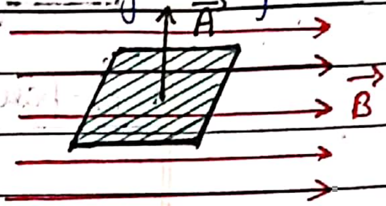
$$\boxed{1 \text{ Weber} = 10^8 \text{ maxwell}}$$

NOTE :→

(1) If the Surface is held PARALLEL to the magnetic field

Here, $\theta = 90°$

$\therefore \phi = BA \cos 90°$ or $\boxed{\phi = 0}$

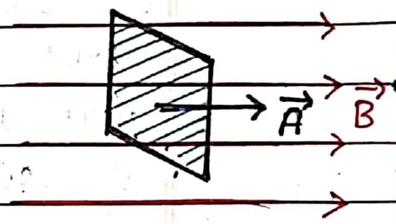


(2) If the surface is held PERPENDICULAR to the magnetic field.

Here, $\theta = 0°$

$\therefore \phi = BA \cos 0°$

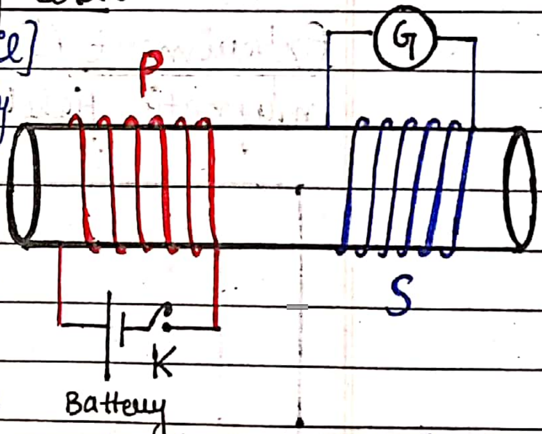
or $\boxed{\phi = \phi_{max} = BA}$



FARADAY'S EXPERIMENTS

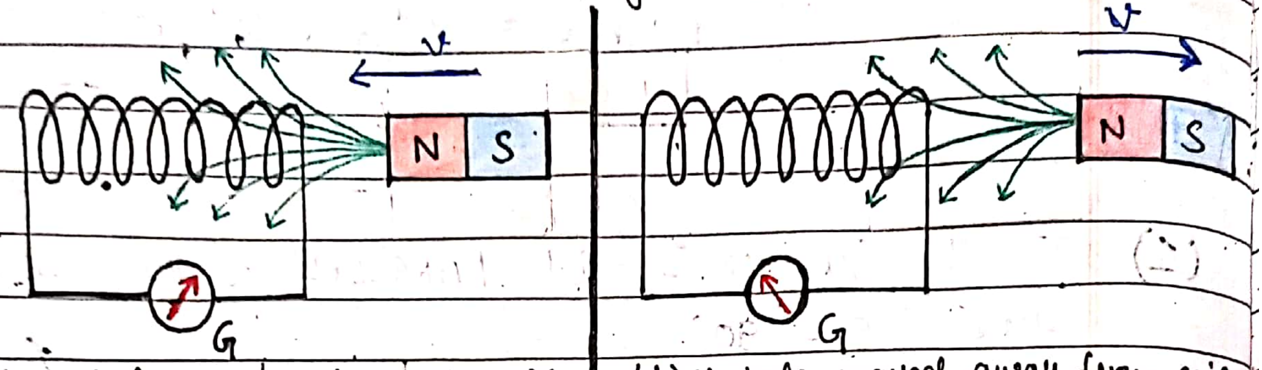
Experiment ① :→ Induced emf by varying current in the neighbouring coil :→

fig: shows two coils P [Primary Coil] & S [Secondary coil] wound independently on a cylindrical support. The coil P is connected to a battery through key (K) and coil S is connected to galvanometer.

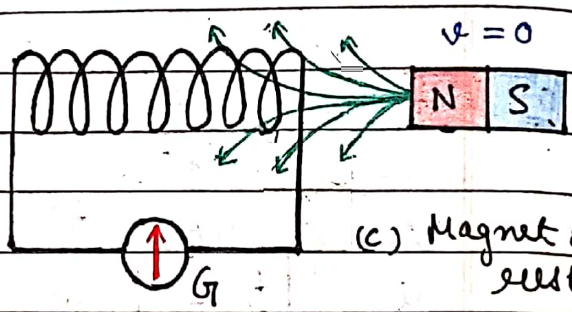


When key 'K' is pressed, galvanometer shows deflection indicating that current is induced in the Secondary coil. If the key K is kept pressed, there is no deflection. When the key is released, galvanometer again shows deflection in opposite direction. Therefore, current is induced in secondary coil by changing the current in primary coil.

Experiment ② :→ Induced emf with a stationary coil and moving magnet



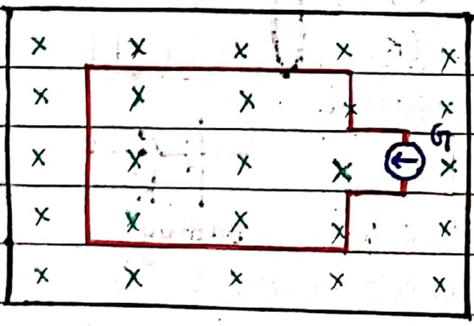
(a) N-pole moved towards coil (b) N-pole moved away from coil



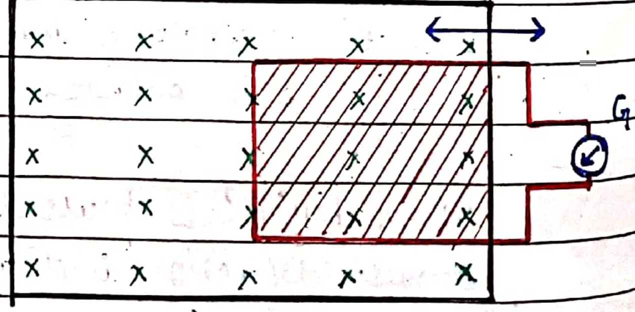
(c) Magnet at rest.

When north pole of a bar magnet is moved towards the coil, the galvanometer shows deflection, indicating that current is induced in the coil. When the magnet is moved away from the coil, galvanometer shows deflection in opposite direction. When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection: current is induced only when the magnet is in motion.

Experiment ③ :→ Induced emf with stationary magnetic field and moving coil :→



(a) coil at rest



(b) coil moved towards right

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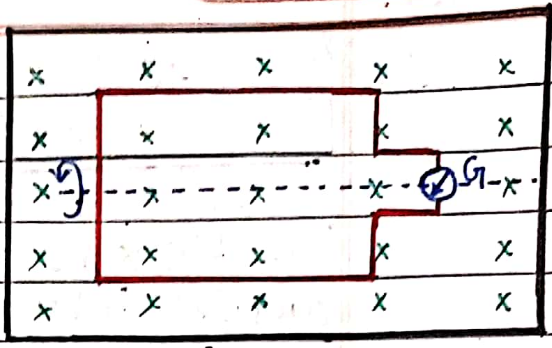
A coil loop is kept in magnetic field and a galvanometer is connected to it [fig (a)]. When the loop is moved to the right, galvanometer shows sudden deflection [fig (b)] which indicates that emf is induced in the coil.



(c) Coil rotated about its axis.

If the coil is rotated about its axis [fig (c)] then also an emf is induced in it.

⇒ NOTE :- Magnetic flux $\boxed{\phi = BA\cos\theta}$ i.e., if either of the three [B, A or θ] changes, there will be a change in magnetic flux.

In all the 3 experiments, magnetic flux is changing.

In exp. ① :- When the current through the Primary coil changes, MAGNETIC FIELD associated with it also changes and thus emf is induced in secondary coil due to change in Magnetic field (B) which changes the flux (ϕ) linked with the coils.

In exp ② :- When the magnet is moved away or towards the coil, the total no. of magnetic field lines (B) linked with the coil changes. This changes the magnetic flux (ϕ) linked with the coil & hence an emf is induced in coil.

In exp ③ :- Magnetic flux changes due to change in the area (A) of the coil in magnetic field when the coil is moved. Hence, emf is induced in it.

Also, when the coil is rotated, 'θ' changes which changes the magnetic flux (ϕ) and hence emf is induced in this case also.

Faster the relative motion between the magnet and the coil, greater is the rate of change of magnetic flux linked with the coil & larger is the induced current set up in the coil. (In exp 2 and 3)

ELECTROMAGNETIC INDUCTION

"The phenomenon of induced emf (and hence induced current) when the magnetic flux linked with a closed circuit is changed."

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

FIRST LAW :→ "Whenever there is a change in magnetic flux linked with closed circuit/loop of wire/coil, an emf is induced in it which lasts as long as the change in flux continues."

SECOND LAW :→ "The magnitude of induced emf is directly proportional to the rate of change of magnetic flux linked with the closed circuit/loop."

i.e., induced emf $\propto \frac{\phi_2 - \phi_1}{t}$

$\phi_1 \Rightarrow$ initial flux
$\phi_2 \Rightarrow$ final flux

$$e \propto \frac{\phi_2 - \phi_1}{t}$$

$$e = -K\frac{(\phi_2 - \phi_1)}{t} \quad \text{--- (1)}$$

-ve sign indicates the opposing nature of 'e' (i.e., emf induced opposes the change in magnetic flux).

In S.I., $k = 1$ ∴ eqn (1) becomes,

$$\boxed{e = -\frac{(\phi_2 - \phi_1)}{t}} \quad (2)$$

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If small flux '$d\phi$' is changed in small time 'dt'
Then,

$$\boxed{e = -\frac{d\phi}{dt}} \quad \text{—— (3)}$$

If there are N turns in the coil, then emf developed in each turn is equal and gets added up. Total induced emf will be

$$e = -\frac{d(N\phi)}{dt}$$

$$\boxed{e = -N\frac{d\phi}{dt}} \quad \text{—— (4)}$$

Q=) The magnetic flux associated with a coil perpendicular to its plane varies as per the relation $\phi = (6t^3 + 3t^2 + 4t - 3)$ weber. If resistance of coil is 5Ω, find the current induced at $t = 3s$.

Sol^n=) Here, $\phi = 6t^3 + 3t^2 + 4t - 3$

as $e = -\frac{d\phi}{dt}$ $\therefore$ $e = -\frac{d(6t^3 + 3t^2 + 4t - 3)}{dt}$

$$e = -[18t^2 + 6t + 4]$$

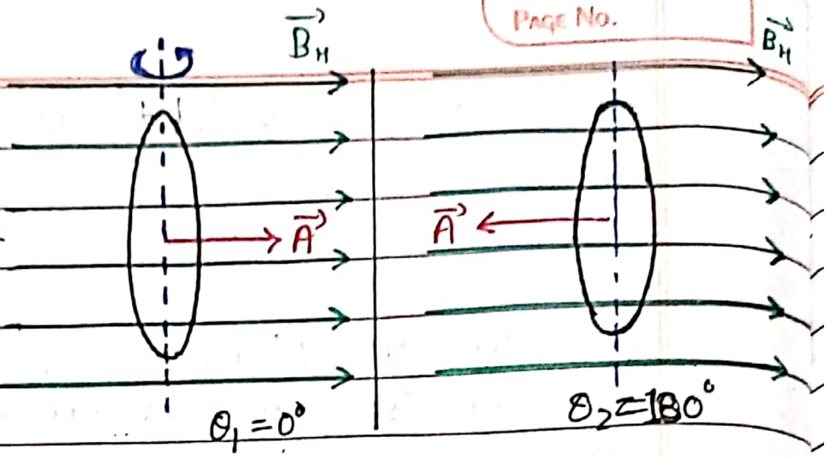
at $t = 3s$,

$$e = -(18 \times (3)^2 + 6 \times 3 + 4)$$

$$\boxed{e = -184\ V}$$

induced current, $I = \frac{|e|}{R} = \frac{184}{5} = 36.8A$

Q=) A circular coil of radius 10 cm, 500 turns and resistance 2Ω is placed with its plane perpendicular to the horizontal component of earth's magnetic field. It is rotated about its vertical diameter through $180°$ in 0.25 s. Estimate the magnitudes of the emf and the current induced in the coil. Horizontal component of earth's magnetic field at the place is 0.3 Gauss.



Sol^n ⇒ Here, $r = 10\,cm = 0.1\,m$

$N = 500$ $R = 2\Omega$

$B_H = 0.3G = 0.3 \times 10^{-4}\,T$

$t = 0.25\,s$

$\theta_1 = 0°$, $\theta_2 = 180°$

$$e = -\frac{(\phi_2 - \phi_1)}{t}$$

$$e = -\frac{[NBA\cos\theta_2 - NBA\cos\theta_1]}{t}$$

$$e = -\frac{NBA}{t}(\cos\theta_2 - \cos\theta_1)$$

$$e = -\frac{500 \times (0.3 \times 10^{-4}) \times \pi(0.1)^2 \,[\cos 180° - \cos 0°]}{0.25}$$

$$e = -\frac{500 \times 3\pi \times 10^{-5} \times 10^{-2} \times (-2)}{25 \times 10^{-2}}$$

$$e = \frac{6\pi}{5} \times 10^{-3} = 3.77 \times 10^{-3}\,V$$

Current induced in the coil,

$$I = \frac{e}{R} = \frac{3.77 \times 10^{-3}}{2} = 1.885 \times 10^{-3}\,A$$

$$\boxed{I = 1.885\,mA}$$

Q ⇒ A coil containing 20 turns of average diameter 0.02 m is placed perpendicular to a magnetic field 1.6×10^4 Tesla. The field changes to 1.8×10^3 T in 4 seconds. A resistance of 15Ω is connected in series with the coil and the resistance of the coil is 5Ω. What is the value of current?

Sol^n ⇒ Here, $N = 20$, $d = 0.02\,m$, $t = 4$ sec.

$B_1 = 1.6 \times 10^4\,T$, $B_2 = 1.8 \times 10^3\,T$

as, $d = 0.02\,m$ ∴ $r = \frac{d}{2} = 0.01\,m$

$A = \pi(0.01)^2$

as,
$$e = -\frac{(\phi_2 - \phi_1)}{t}$$

$$e = -\frac{(NB_2A\cos\theta - NB_1A\cos\theta)}{t}$$

$$e = -NA\cos\theta \frac{(B_2 - B_1)}{t}$$

$$e = -\frac{20 \times \pi (0.01)^2 \cos 0^\circ [1.8 \times 10^{-3} - 1.6 \times 10^{-4}]}{4}$$

$$e = -\frac{20\pi \times 10^{-4} [(0.18 - 1.6) \times 10^{-4}]}{4}$$

$$e = -5\pi \times (-1.42)$$

$$e = 22.305 \text{ V}$$

$R_{total} = 15 + 5 = 20\Omega$ (as both resistances are in series)

Induced current, $I = \frac{e}{R_{tot}} = \frac{22.305}{20}$

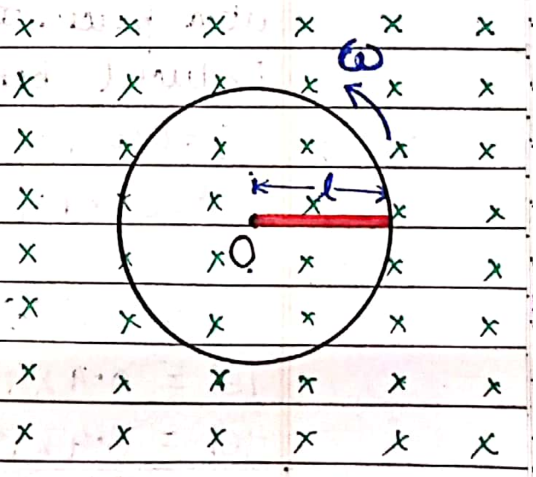
$$\boxed{I = 1.115 A}$$

Q=) A conducting rod of length 'l' with one end pivoted is rotated with a uniform angular speed 'ω' in a uniform magnetic field B in a plane normal to the field. Find the expression for the emf induced across the ends of the rod.

Solⁿ=) Let the rod completes one round in time $t = T$

flux linked $(\phi) = BA$ area swept

$$\phi = B\pi l^2$$



Induced emf $(e) = -\frac{\text{change in flux}}{\text{time}}$

$$e = -\frac{(B\pi l^2 - 0)}{T}$$

$e =$

$$e = -\frac{B\pi l^2}{T} = \frac{-B\pi l^2}{2\pi/\omega}$$ [as $\omega = \frac{2\pi}{T}$ ∴ $T = \frac{2\pi}{\omega}$]

∴ $\boxed{e = -\frac{1}{2}Bl^2\omega}$ —— Ans.

as $\omega = 2\pi\nu$ ∴ $e = -\frac{1}{2}Bl^2(2\pi\nu)$

$$\boxed{e = -B\pi l^2\nu}$$

Q => A metal rod of length 1 m is rotated about one of its ends in a plane perpendicular to a uniform magnetic field of 2.5×10^{-3} Wb/m². If it makes 30 revolutions per second (rps). Find emf induced between its ends.

Solⁿ => Here, $l = 1$ m, $B = 2.5 \times 10^{-3}$ T, $\nu = 30$ rps

as $e = -B\pi l^2\nu$

$e = -2.5 \times 10^{-3} \times 3.14 \times 1^2 \times 30$

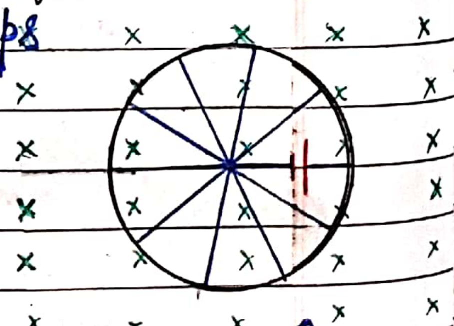
$e = -0.23$ V

-ve sign indicates that induced emf opposes the change in flux.

Q => A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rpm in a plane normal to the horizontal component of earth's magnetic field (B_H) at a place. If $B_H = 0.40$ G at that place, what is the emf induced between the axle & the rim of the wheel.

Solⁿ => $l = 0.5$ m, $\nu = 120$ rpm $= \frac{120}{60} = 2$ rps

$B = 0.4$ G $= 0.4 \times 10^{-4}$ T



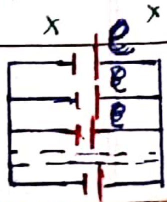
$|e| = B\pi l^2\nu$

$|e| = 0.4 \times 10^{-4} \times 3.14 \times (0.5)^2 \times 2$

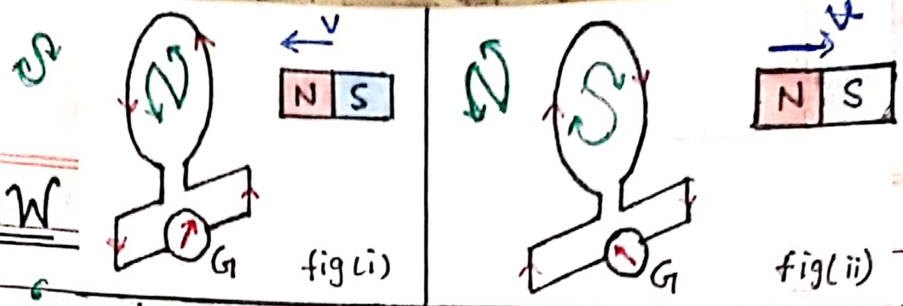
$|e| = 0.4 \times 10^{-4} \times 0.25 \times 10^{-4} \times 6.28$

$\boxed{|e| = 6.28 \times 10^{-5} \text{ V}}$

=> As all the ten spokes are connected in parallel, so $\boxed{E_{tot} = e}$



LENZ'S LAW



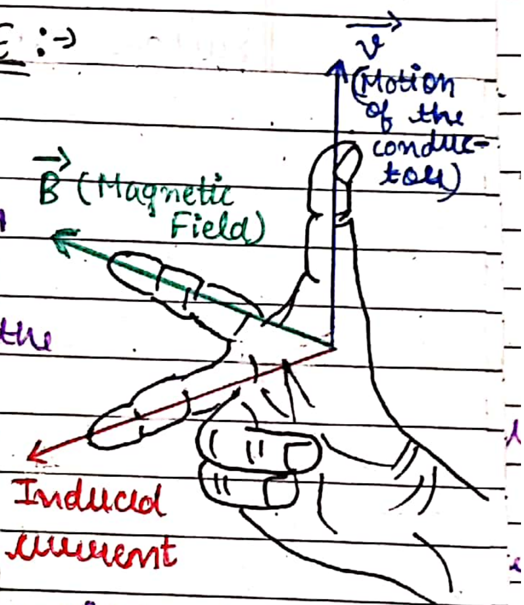
It states that – 'Induced current produced in a circuit always flows in such a direction that it opposes the change (or the cause) by the virtue of which it is produced.'

Direction of the Induced Current :→

The direction of the induced current can be found either by using – FLEMING'S RIGHT HAND RULE or by using LENZ'S LAW.

FLEMINGS RIGHT HAND RULE :→

Stretch the thumb, fore-finger and central finger of the right hand in such a way that they are mutually perpendicular to each other. Keep the fore finger in the direction of magnetic field, thumb in the direction of motion of conductor then the direction in which the central finger points gives the direction of the Induced current.



Q⇒ Verify that the 'Lenz's Rule is in accordance with Law of Conservation of energy.'

Ans⇒ When a magnet is moved towards or away from a closed coil, the induced current always opposes the motion of the magnet, as predicted by Lenz's Law. For example, when the north pole of a magnet is brought closer to a coil [fig(i)], its face towards the magnet develops north polarity and thus repels north pole of the magnet. Work has to be done in moving the magnet closer to the coil against this force of repulsion. Similarly, when the north pole of the magnet is moved away from the coil [fig(ii)], its face towards the magnet develops south polarity and thus attracts the north pole of the magnet. Here, work has to be done in moving the magnet away from the coil against this force of attraction. It is this work done against the force of repulsion or attraction that appears as electric energy in the form of induced current.

Suppose that the Lenz's law is not valid. Then the induced current flows through the coil in a direction opposite to one dictated by Lenz's Law. The resulting force on the magnet makes it move faster and faster, i.e., the magnet gains speed and hence kinetic energy without expending an equivalent amount of energy. This set up a perpetual motion machine, violating the law of conservation of energy. Thus, Lenz's Law is valid and is a consequence of the law of conservation of energy.

MOTIONAL EMF

'The emf induced across the ends of a conductor due to its motion in a magnetic field is called Motional emf.'

Let a rectangular coil PQRS is moving out of a uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$ as shown in fig. The Magnetic field is directed normally into the

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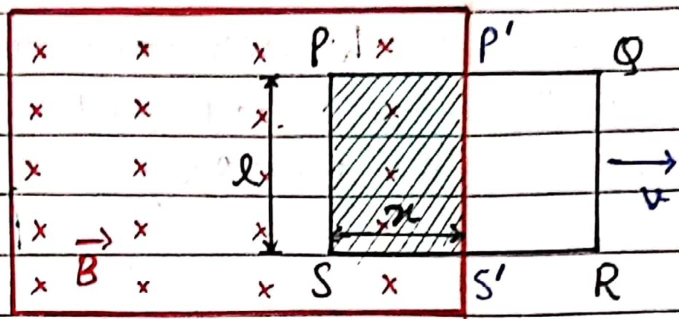
plane of paper. As the loop slides towards right, the area of rectangular coil inside the M.F. decreases. This decreases the magnetic flux linked with the coil. Hence an emf is induced across the ends of the coil PQRS.



Let $PS = l$ and at any instant 't', $PP' = SS' = x$

Now, at that instant, magnetic flux linked with the loop $(\phi) = BA = B \times (lx)$

$\therefore$ emf induced $(e) = -\dfrac{d\phi}{dt} = \dfrac{-d(Blx)}{dt}$

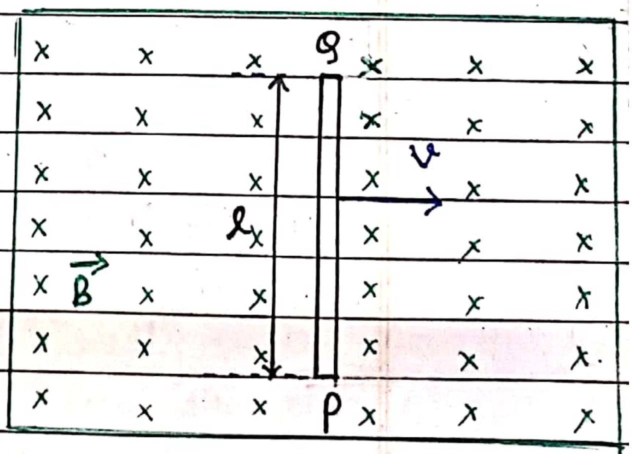
$$e = -Bl\frac{dx}{dt}$$

$$\boxed{e = Blv} \longrightarrow (1)$$

where $\dfrac{dx}{dt} = -v$, because the velocity is in the decreasing direction of x.

MOTIONAL EMF from LORENTZ FORCE

Consider a conductor PQ is placed in a magnetic field $\vec{B}$ perpendicular to it as shown. Let the conductor is moved with constant velocity '$\vec{v}$' towards right side.

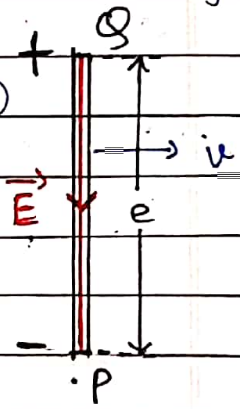


When the conductor moves, the free electrons present in it also move with same velocity $\vec{v}$ in M.F. $\vec{B}$. As a result, the Lorentz force acts on free electrons in the direction from Q to P (by Fleming's Left Hand Rule) and is given by the relation,

$$\vec{F}_m = -e(\vec{v} \times \vec{B}) \quad \text{—— (1)}$$

Due to Lorentz force free e^-s will move towards P. A −ve charge accumulates at P and +ve charge at Q. An electric field '$\vec{E}$' is set up in the conductor from Q to P as shown in fig. below. This field exerts a force on the free e^-s given by :-

$$\vec{F}_e = -e\vec{E} \quad \text{—— (2)}$$



The accumulation of charges at the two ends continues till these two forces balance each other, i.e.,

$$|\vec{F}_m| = |\vec{F}_e|$$

$$|-e(\vec{v} \times \vec{B})| = |-e\vec{E}|$$

$$vB\sin 90^\circ = E$$

$$vB = E \quad \text{—— (3)}$$

The potential difference b/w the ends Q and P is

$$V = El$$

$$V = (vB)l \quad \text{—— (4)}$$

Clearly, it is the magnetic force on the moving free e^-s that maintains the Potential difference & produces the emf,

$$\boxed{e = Blv} \quad \text{—— (5)}$$

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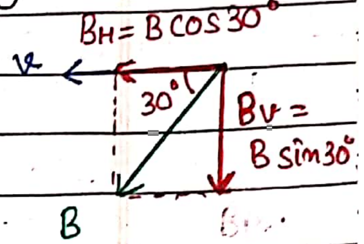
Q⇒ A metre gauge train is running due north with a constant speed of 90 km/h on a horizontal track. If earth's magnetic field at that place is 0·3 Gauss, calculate the emf induced across the ends of the train of length 1 m. Given δ (angle of dip) at that place is 30°



Sol^n ⇒ $v = 90\ km/h = 90 \times \frac{5}{18} = 25\ m/s$

$B = 0{\cdot}3\ G = 0{\cdot}3 \times 10^{-4}\ T$

angle of dip $\delta = 30°$ $\quad l = 1\ m$

Here, $e = (B_v)\, l\, v$

$e = (B\sin\delta)\, l\, v$

$e = 0{\cdot}3 \times 10^{-4} \times \sin 30 \times 1 \times 25$

$e = 3 \times \frac{1}{2} \times 25 \times 10^{-5}$

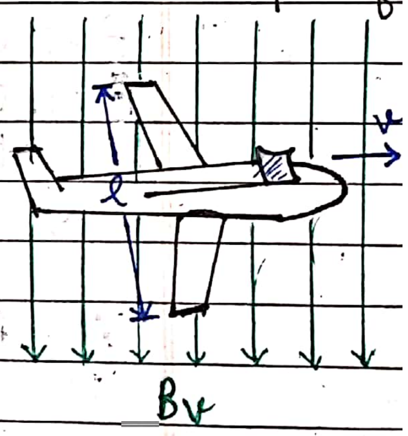
$\boxed{e = 3{\cdot}75 \times 10^{-4}\ V}$

Q⇒ An aircraft with a wing span of 40 m flies with a speed of 1080 kmh^{-1} in the eastward direction at a constant altitude where the vertical component of earth's magnetic field $1{\cdot}75 \times 10^{-5}$ T. Find the emf induced across the tips of its wings.

Sol^n ⇒ Here, $l = 40\ m$, $B_v = 1{\cdot}75 \times 10^{-5}\ T$

$v = 1080\ km/h = 1080 \times \frac{5}{18}\ m/s$

$v = 300\ m/s$



So, emf induced b/w the tips of wings

$e = B_v\, l\, v$

$e = 1{\cdot}75 \times 10^{-5}\ T \times 40 \times 300$

$\boxed{e = 0{\cdot}21\ V}$

Q⇒ A conductor of length 1·0 m falls freely under gravity from a height of 10 m so that it cuts the lines of force of the horizontal component of earth's magnetic field of $3 \times 10^{-5}\ Wb\,m^{-2}$. Find the emf induced in conductor.

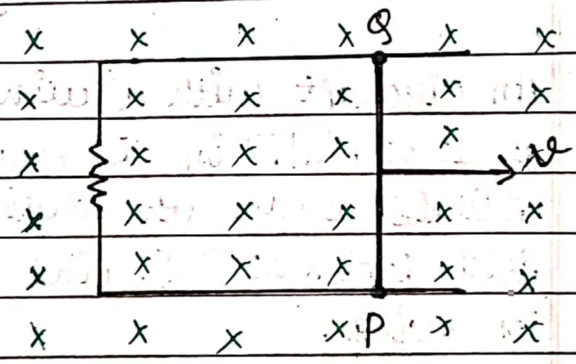
Sol^n ⇒ The velocity 'v' attained by the conductor as it falls through a height of 10m is given by

$$v^2 = u^2 + 2gs = 0 + (2 \times 9.8 \times 10)$$
$$v^2 = 4 \times 49$$
$$v = 2 \times 7 = 14\,m/s$$

∴ Induced emf,

$$e = B_H l v = 3 \times 10^{-5} \times 1 \times 14$$
$$\boxed{e = 4.2 \times 10^{-4}\,V}$$

Q ⇒ A 0.5m long metal rod PQ completes the circuit as shown in fig. The area of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the total circuit is 3Ω, calculate the force needed to move the rod, in the direction as indicated with a constant speed of 2m/s.



Sol^n ⇒ as $e = Blv$

∴ Induced current

$$I = \frac{e}{R} = \frac{Blv}{R}$$

$$F = IlB \sin 90°$$

$$F = \left(\frac{Blv}{R}\right) \times l \times B = \frac{B^2 l^2 v}{R}$$

Here, $l = 0.5\,m$, $B = 0.15\,T$, $R = 2\Omega$, $v = 2\,m/s$

$$\therefore F = \frac{(0.15)^2 \times (0.5)^2 \times 2}{3} = \frac{225 \times 25 \times 2 \times 10^{-6}}{3}$$

$$F = 75 \times 5 \times 10^{-5}$$
$$F = 375 \times 10^{-5}$$
$$\boxed{F = 3.75 \times 10^{-3}\,N}$$

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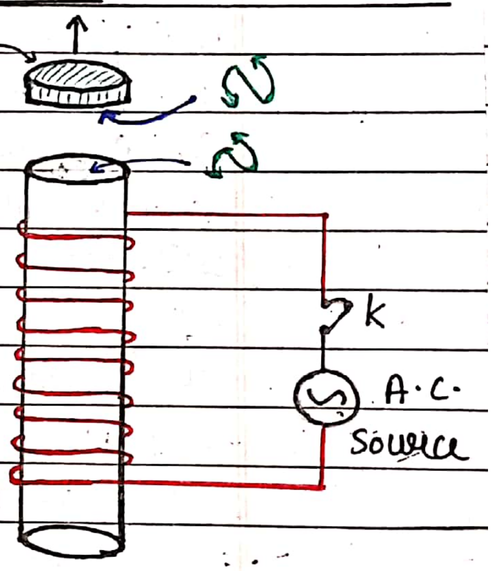
EDDY CURRENTS

Currents can be induced, not only in conducting coils, but also in conducting sheets or blocks.

"When the magnetic flux linked with a solid metallic conductor changes, the currents are induced in the conductor in the form of closed loops which look like eddies or whirlpools known as Eddy currents.

NOTE :-> Eddy currents also oppose the change in the magnetic flux, so there direction is given by Lenz's Law.

EXPERIMENTAL DEMONSTRATION OF EDDY CURRENTS :

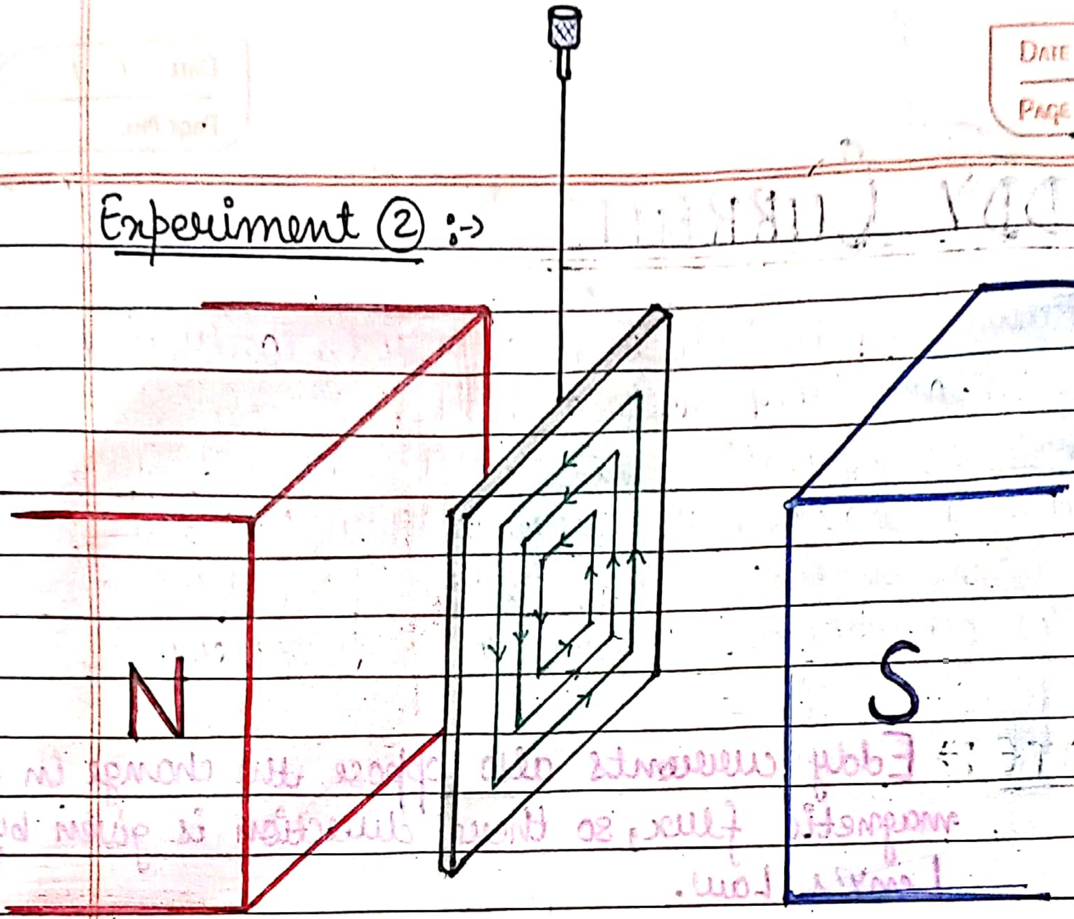


Experiment ① :->

Introduce a soft iron core inside a solenoid and connect it to an A.C. source. When a metallic disk is placed over the soft iron core and the circuit is switched on, the metallic disk is thrown up into the air.

When circuit is switched, the current grows due to which the magnetic field and hence magnetic flux through the disk also increases which set up the induce current in the disk which tends to flow in such a manner which opposes the growth of magnetic flux through it due to which the similar poles are produced in the lower space of the disk and the upper face of the iron core as a result of which the disk is thrown up.

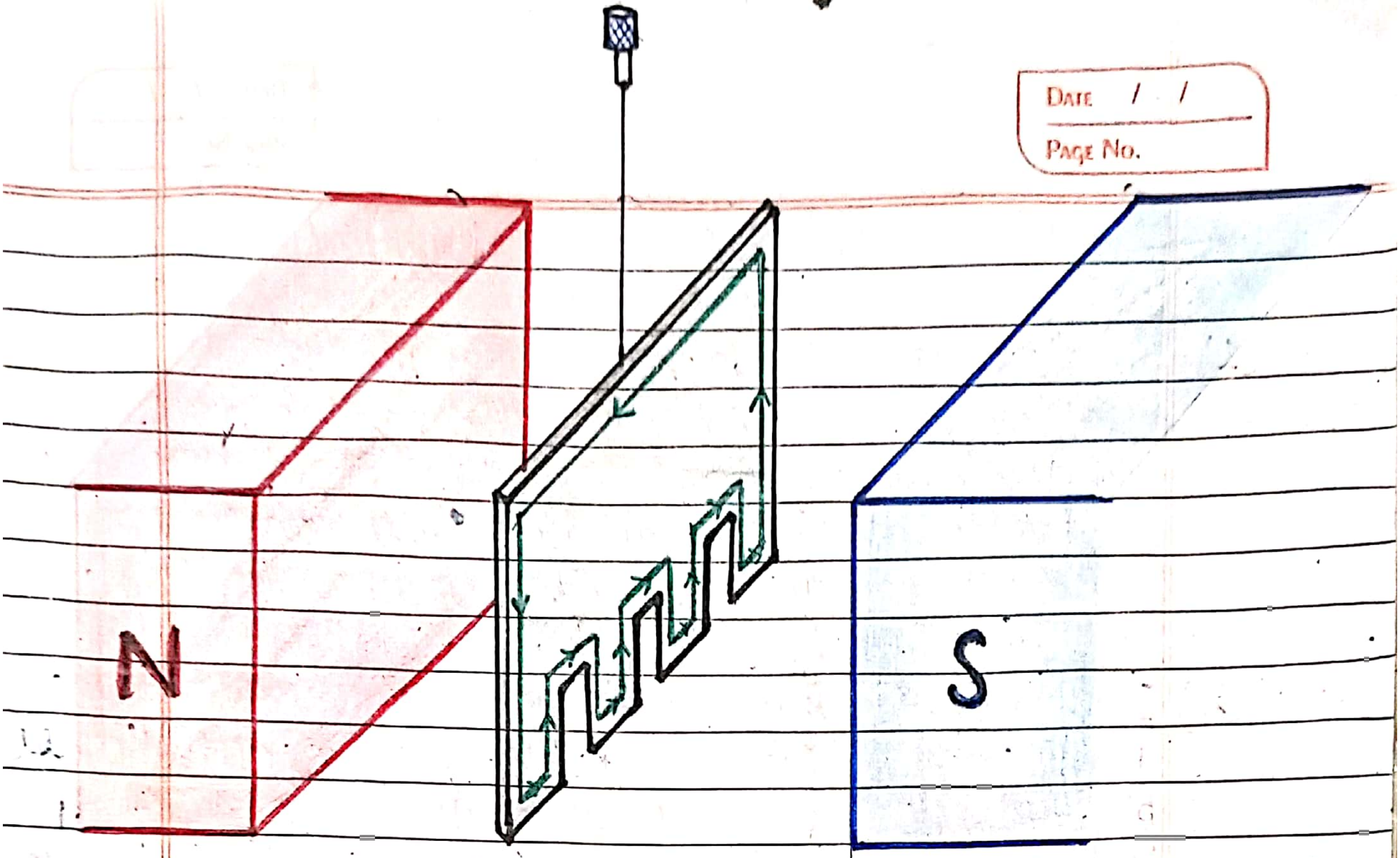
Experiment ② :->



Suspend a flat metallic plate (copper or aluminium) b/w the poles of an electromagnet. Initially when the field is off and the plate is allowed to oscillate freely. It can oscillate for considerably long time and as soon as the magnetic field is switched on, the oscillations die out quickly.

When the plate leaves or enters the magnetic field during its vibrations or oscillations, the changing magnetic flux linked with it will cause the induce currents (eddy currents) to produce in it which opposes the cause of their production i.e., increase or decrease in magnetic flux and hence the motion / oscillations of the sheet. Thus, the oscillations die out soon.

Experiment ③ :-> If the experiment ② is repeated by using a metallic plate having slots, it oscillates for considerably long time as compared to the previous case.

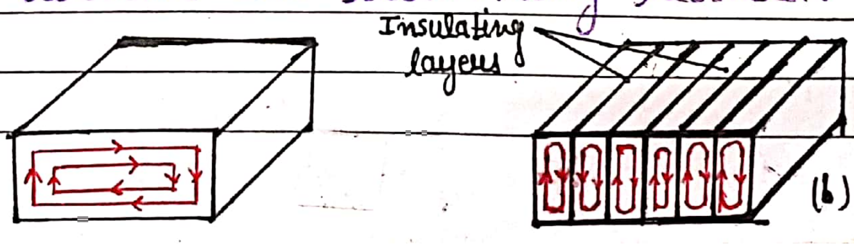


This is because loop has much larger paths for e^-s to travel. Larger path offers more resistance to the e^-s and so the magnitude of the eddy currents produced in this case will be very less. Hence, their opposing nature will also be less. Thus, plate oscillates for long time.

NOTE :- Eddy currents produced in iron cores of the rotating armatures of electric motors and dynamos and also in the cores of transformers cause unnecessary heating and wastage of power, which may even damage the insulation of coils.

Minimisation of Eddy Currents :- The eddy currents can be reduced by using **LAMINATED CORE** which instead of single solid mass consists of thin sheets of metals, insulated from each other by a thin layer of varnish. The insulation b/w the sheets offers high resistance to the induced emf and the eddy currents are substantially reduced.

(a) Solid core



(b) Laminated core

Applications of Eddy Currents :-

(1) Dead-Beat Galvanometer ⇒ By winding the coil of a galvanometer on a metallic frame its oscillations can be damped very quickly. Such a galvanometer is called Dead Beat Galvanometer. It is due to the production of eddy currents in the metallic frame.

(2) Induction Furnace ⇒ In induction furnace, the metal to be heated is placed in rapidly varying magnetic field produced by high frequency alternating current. Strong eddy currents are set up in the metal which produce so much heat that the metal melts. This process is used in extracting metal from its ores.

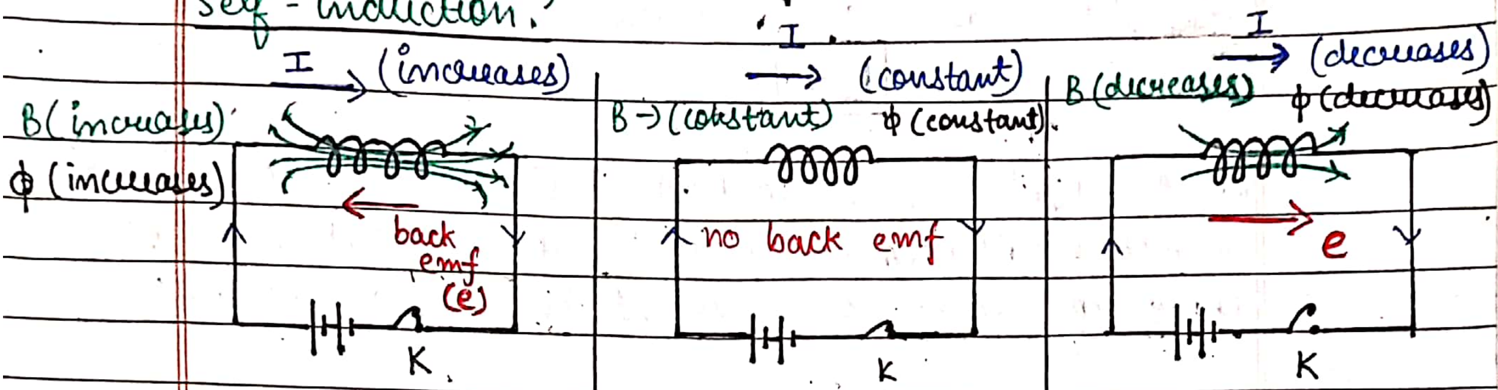
(3) Diathermy ⇒ Eddy currents can be used for deep heat treatment of human tissues. This is called diathermy.

(4) Electric Brake ⇒ A metallic drum is coupled to the wheels of the train so that when the train runs the drum also rotates. In order to stop the train a strong magnetic field is applied to the rotating drum. The large eddy currents are produced in the drum which opposes the motion of the drum and hence reduces the speed of the train.

⇒ Besides these, the eddy currents are used in electromagnetic damping, induction motors, speedometer, etc.

SELF-INDUCTION

"The phenomenon of production of an opposing induced emf in a coil due to the change in current or magnetic flux linked with the coil itself is called self-induction."



(a) Circuit is just closed

(b) Circuit remains closed

(c) circuit is opened.

COEFFICIENT OF SELF-INDUCTION

At any instant, $\phi \propto I$ [$\phi \rightarrow$ magnetic flux linked with coil; $I \rightarrow$ current through coil]

or $\boxed{\phi = LI}$ —— (1)

$L \Rightarrow$ constant called COEFFICIENT OF SELF-INDUCTION or SELF INDUCTANCE OF THE COIL.

If $I = 1$, then from eq^n (1), $\boxed{L = \phi}$

∴ "Coefficient of self Induction/Self Inductance of a coil is numerically equal to the magnetic flux linked with the coil if a unit current flows through it."

If e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt} \qquad [\text{as } \phi = LI]$$

$$\boxed{e = -L\frac{dI}{dt}} \quad \text{—— (2)}$$

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In magnitude, $\boxed{e = L\frac{dI}{dt}}$

$$\therefore \quad L = \frac{e}{\frac{dI}{dt}} \quad \text{——— (3)}$$

If $\frac{dI}{dt} = 1$, then from eq^n (3)

$$\boxed{L = e}$$

∴ Coefficient of self-induction / self inductance of a coil is numerically equal to the emf induced in it when rate of change of current in the coil is unity.

Units of Self-Inductance :→ as from eq^n (3)

$$L = \frac{e}{\frac{dI}{dt}}$$

∴ SI Unit of L $= \frac{\text{Volt}}{A/s} = VsA^{-1} = $ Henry (H)

as, $L = \frac{e}{dI/dt}$, if $e = 1V$ and $\frac{dI}{dt} = 1 A/s$, then

$$\boxed{L = 1 \text{ Henry}}$$

1 Henry :→ Thus, the self inductance of a coil is said to be 1 HENRY if a current changing at the rate of 1 A/s induces an emf of 1 V in it.

Dimensions of Self Inductance

from eq^n (1) $L = \frac{\phi}{I} = \frac{BA}{I} = \frac{F}{qV\sin\theta} \cdot \frac{A}{I}$ $[\because F = qvB\sin\theta]$

$\therefore$ Dimensions of $L = \frac{MLT^{-2}}{C \cdot LT^{-1}} \cdot \frac{L^2}{A} = \frac{ML^2T^{-2}}{A \cdot A}$

" " " $= [ML^2T^{-2}A^{-2}]$ $\quad$ [as $q/t = I$ $\therefore CT^{-1} = A$]

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⇒ **NOTE :→** Self - Induction is also known as INERTIA OF ELECTRICITY.

Self - Induction is the property of an inductor to induce a current opposite to the direction of the emf applied when there is a change in magnetic flux across the inductor i.e; self - inductance opposes the growth and decay of the current through an inductor. Whereas speaking about the inertia, it is defined as a resistance in change in state of rest or motion of the object. Hence, we can say that self - inductance is the inertia of electricity

Q⇒ If the current changing at a rate of 4 A/s induces an emf of 20 mV in a solenoid, find the self inductance of the solenoid.

Sol^n⇒ Here, $\frac{dI}{dt} = 4\,A/s$, $|e| = 20\,mV = 20 \times 10^{-3}\,V$

as $|e| = L\frac{dI}{dt}$ ⇒ $L = \frac{e}{\frac{dI}{dt}}$

$\therefore$ $L = \frac{20 \times 10^{-3}}{4} = 5 \times 10^{-3}\,H$ or $\boxed{L = 5\,mH}$

Q⇒ The self - inductance of a coil having 200 turns is 10 mH. Compute the Total flux linked with the coil. Also determine the magnetic flux through the cross section of the coil corresponding to a current of 4 mA.

Sol^n⇒ Here, $L = 10\,mH = 10 \times 10^{-3}\,H = 10^{-2}\,H$, $N = 200$

$I = 4\,mA = 4 \times 10^{-3}\,A$

as $\phi = LI$

$\therefore$ Total flux linked with 200 turns $= 10^{-2} \times 4 \times 10^{-3}$
$= 4 \times 10^{-5}\,Wb$

flux through cross section = flux linked with 1 turn
$= \frac{4 \times 10^{-5}}{200}$
$= 2 \times 10^{-7}\,Wb$

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Q⇒ A 5 Henry inductor carries a steady current of 2A. How can a 50V self induced emf be produced in the inductor.

Solⁿ ⇒ Here, $L = 5H$, $I = 2A$, $e = 50V$

Let the current is switched off in time dt.

then $dI = 2$

as $|e| = L\frac{dI}{dt}$

$50 = \frac{5 \times 2}{dt}$ ⇒ $dt = \frac{10}{50}$

⇒ $\boxed{dt = 0.2s}$

∴ 50V emf can be induced by changing the current from 2 A to 0 A in 0.2 s.

Q⇒ A coil has a self inductance of 10mH. What is the maximum magnitude of the emf induced in the inductor when a current of $I = 0.1 \sin 200t$ A is passed through it.

Solⁿ ⇒ $L = 10mH = 10 \times 10^{-3} H$ $I = 0.1 \sin 200t$

$e_0 (e_{max}) = ?$

$e = L\frac{dI}{dt}$

$e = (10 \times 10^{-3}) \frac{d}{dt}(0.1 \sin 200t)$

$e = 10 \times 10^{-3} [0.1 \cos 200t \times (200)]$

$e = 200 \times 10^{-3} \cos 200t$

$e = 0.2 \cos 200t$

for above value to be maximum

$\cos 200t = \text{maximum} = +1$

i.e., $\boxed{e = e_0 = 0.2V}$ Ans.

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ENERGY STORED IN AN INDUCTOR

When an inductor is connected to a source of emf, the external source has to expend energy in building up the current through the inductor against induced emf. This energy is stored in the inductor as MAGNETIC FIELD ENERGY.

Let at any instant current in the inductor of self inductance 'L' be 'I' and rate of change of current is $\frac{dI}{dt}$ at that instant. Then emf induced is,

$$e = L\frac{dI}{dt} \quad \text{(in magnitude)}$$

Small work done by source of emf in small time dt

$$dW = P\,dt = eI\,dt \quad [\text{as } P = eI]$$

$$dW = \left(L\frac{dI}{dt}\right) I\,dt$$

$$dW = LI\,dI$$

∴ Total work done to raise the current from 0 to I is

$$W = \int_0^I LI\,dI$$

$$W = L\int_0^I I\,dI$$

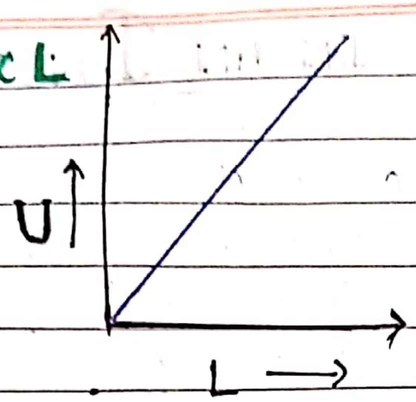
$$W = L\left[\frac{I^2}{2}\right]_0^I$$

$$W = \frac{1}{2}LI^2$$

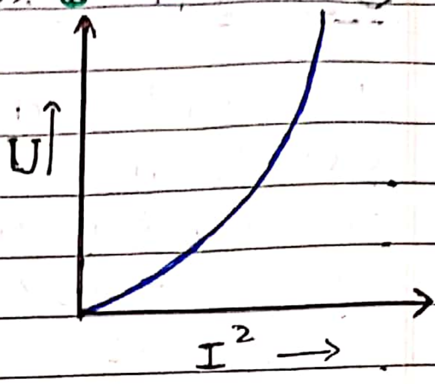
This work done is stored in the inductor in the form of Magnetic field Energy.

∴ energy stored in inductor $\boxed{U = \frac{1}{2}LI^2}$

(i) $U \propto L$



(ii) $U \propto I^2$



SELF INDUCTANCE OF A LONG SOLENOID

Consider a long solenoid of length $= l$
Area of cross section $= A$
no. of turns per unit length $= n$
Let I current is passed through it, then magnetic field inside the solenoid is,

$$B = \mu_0 n I \quad \text{——— (1)}$$

Magnetic flux linked through each turn of solenoid

$$= B \times A = \mu_0 n I A \quad \text{——— (2)}$$

Total magnetic flux linked with solenoid

= magnetic flux through each turn × total no. of turns in the solenoid

$$\phi = (\mu_0 n I A) \times (n l) = \mu_0 n^2 l I A \quad \text{——— (3)}$$

If 'L' is the self inductance of the solenoid then

$$\phi = L I \quad \text{——— (4)}$$

∴ from eq^n (3) & (4)

$$L \cancel{I} = \mu_0 n^2 l \cancel{I} A$$

$$\boxed{L = \mu_0 n^2 l A} \quad \text{——— (5)}$$

NOTE :→ Factors on which self - Inductance depends :→ from eqⁿ ⑥

(i) No. of turns :→ $L \propto N^2$

(ii) Area of cross section :→ $L \propto A$

(iii) Permeability of the core material

If $N \Rightarrow$ total no. of turns then

$$N = nl \quad \therefore \quad n = \frac{N}{l}$$

$\therefore$ from eqⁿ ⑤

$$L = \mu_0 \left(\frac{N}{l}\right)^2 l \cdot A$$

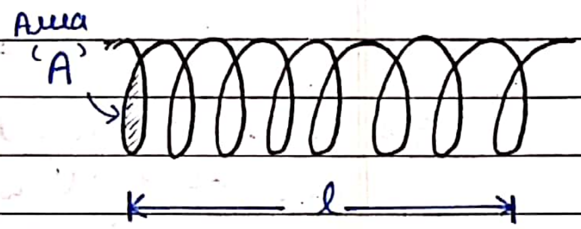
$$\boxed{L = \frac{\mu_0 N^2 A}{l}} \quad \text{—— ⑥}$$

ENERGY STORED IN A SOLENOID

Self inductance of a long solenoid is

$$L = \mu_0 n^2 l A \quad \text{—— ①}$$

Magnetic field inside a solenoid is $B = \mu_0 n I$



$$\therefore I = \frac{B}{\mu_0 n} \quad \text{—— ②}$$

as we obtained, energy stored in an inductor is given by —

$$U = \frac{1}{2} L I^2 \quad \text{—— ③}$$

$\therefore$ from eqⁿ ①, ② and ③

$$U = \frac{1}{2} [\mu_0 n^2 l A] \left[\frac{B}{\mu_0 n}\right]^2$$

$$\boxed{U = \frac{1}{2} \frac{B^2}{\mu_0} A l} \quad \text{—— ④}$$

Energy density = Energy stored per unit volume

$$\bar{U} = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}\frac{B^2}{\mu_0}\cancel{Al}}{\cancel{Al}}$$

$$\boxed{\bar{U} = \frac{1}{2} \frac{B^2}{\mu_0}} \quad \text{—— ⑤}$$

Q.) An air cored solenoid with length 30 cm, area of cross-section 25 cm^2 and no. of turns 500, carries a current of 2·5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit?

Solⁿ.) $l = 30\,cm = 0.3\,m$, $A = 25\,cm^2 = 25 \times 10^{-4}\,m^2$

$N = 500$, $dt = 10^{-3}\,s$, $dI = 0 - 2.5 = -2.5\,A$

$$\text{Back emf } (e) = -L\frac{dI}{dt} = -\frac{\mu_0 N^2 A}{l}\frac{dI}{dt}$$

$$e = \frac{-4\pi \times 10^{-7} \times (500)^2 \times 2.5 \times 10^{-4} \times (-2.5)}{6.3 \times 10^{-3}}$$

$$\boxed{e = 6.542\,V}$$

Q.) A coil has an inductance of 5 Henry and resistance 20Ω. An emf of 100 V is applied to it. What is the energy stored in magnetic field when the current had reached its final steady value?

Solⁿ.) $V = 100\,V$ $R = 20\,\Omega$ $L = 5\,H$

$$I = \frac{V}{R} = \frac{100}{20} = 5A$$

$$U = \frac{1}{2}LI^2 = \frac{1}{2} \times 5 \times 25$$

$$\boxed{U = 62.5\,J}$$

Q.) A 100 μF capacitor is charged with a 50 V source supply. Then source supply is removed and the capacitor is connected across an inductance, as a result of which 5A current flows through the inductance. Calculate the value of inductance.

Solⁿ Here $C = 100\,\mu F = 100 \times 10^{-6}\,F = 10^{-4}\,F$

$V = 50$ Volt , $I = 5A$, $L = ?$

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When the capacitor is charged with 50 V supply, energy gets stored in it which is given by

$$U = \frac{1}{2}CV^2$$

When this charged capacitor is connected across the inductor, then

Energy stored in the capacitor = Energy stored in the inductor

$$\frac{1}{2}CV^2 = \frac{1}{2}LI^2$$

$$L = \frac{CV^2}{I^2} = \frac{10^{-4}\times(50)^2}{5^2} = \frac{10^{-4}\times 25\times 10^2}{25}$$

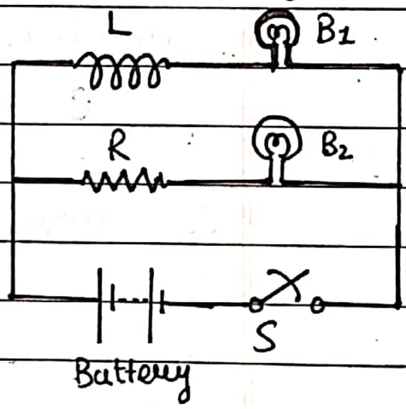
$$\boxed{L = 10^{-2} = 0\cdot01\,H}$$ Ans.

Q⇒ Fig. shows an inductor L and a resistor R connected in parallel to a battery through a switch. The resistance R is the same as that of the coil that makes L. Two identical bulbs are put in each arm of the circuit.

(i) Which of the bulb lights up bright when S is closed?

(ii) Will the two bulbs be equally bright after some time?

Give reason for your answer.

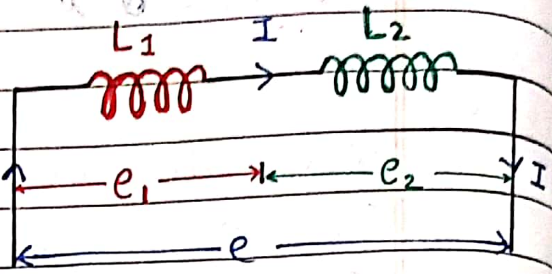


Ans⇒(i) The bulb B_2 lights up earlier than the bulb B_1. This is because when S is closed, an induced current is set up in the inductor L which opposes the growth of current in L-arm and hence bulb B_1 is less bright than B_2.

(ii) After some time, current in both the arms becomes steady. Hence, no back emf is produced in the inductor L. So, the two bulbs will be equally bright.

GROUPING OF INDUCTORS

(1) Inductors in Series :→



Let two inductors of self inductance L_1 & L_2 are connected in series as shown.

If $\frac{dI}{dt}$ is the rate of change of current then

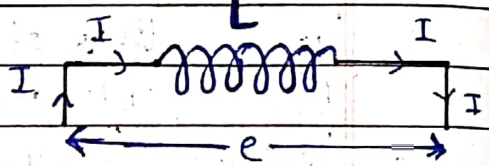
emf induced in L_1, $e_1 = -L_1 \frac{dI}{dt}$ —— (1)

emf induced in L_2, $e_2 = -L_2 \frac{dI}{dt}$ —— (2)

Total emf induced $e = e_1 + e_2$

$$e = -L_1 \frac{dI}{dt} + \left(-L_2 \frac{dI}{dt}\right) \quad \text{—— (3)}$$

If 'L' is the equivalent self inductance then



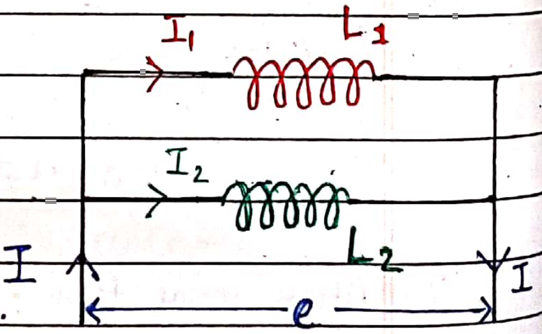
$$e = -L \frac{dI}{dt} \quad \text{—— (4)}$$

∴ from eqⁿ (3) & (4)

$$-L \frac{dI}{dt} = -(L_1 + L_2) \frac{dI}{dt}$$

$$\boxed{L = L_1 + L_2} \quad \text{—— (4)}$$

(2) Inductors in Parallel :→



Let two inductors of self inductance L_1 & L_2 are connected in parallel as shown.

If, $\frac{dI_1}{dt} \Rightarrow$ rate of change of current in L_1

& $\frac{dI_2}{dt} \Rightarrow$ rate of change of current in L_2

Let e is the emf induced

For ||el combination, induced emf across the combination is equal to the induced emf across each inductor.

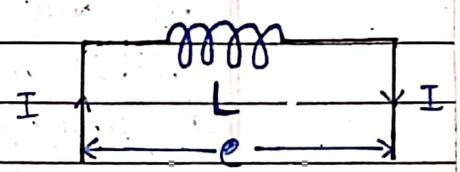
$$\therefore \quad e = -L_1 \frac{dI_1}{dt} \quad \text{and} \quad e = -L_2 \frac{dI}{dt}$$

$$\Rightarrow \quad \frac{dI_1}{dt} = \frac{-e}{L_1} \quad \text{—— (1)} \quad \text{and} \quad \frac{dI_2}{dt} = \frac{-e}{L_2} \quad \text{—— (2)}$$

as $I = I_1 + I_2 \quad \therefore \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$ —— (3)

If 'L' is the equivalent inductance, then

$$e = -L \frac{dI}{dt}$$



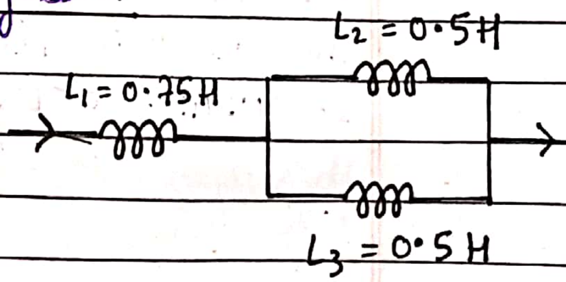
$$\frac{dI}{dt} = \frac{-e}{L} \quad \text{—— (4)}$$

from eqⁿ (3) & (4) $\quad \frac{-e}{L} = \frac{-e}{L_1} + \left(\frac{-e}{L_2}\right)$

$$\boxed{\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}} \quad \text{—— (5)}$$

⇒ **NOTE :->** In both the above cases, the inductors are so far apart that their mutual inductance is negligible.

Q⇒ Three inductors are connected as shown. Find the equivalent inductance.

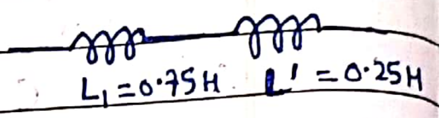


Sol^n:- The equivalent inductance L' of L_2 & L_3 is

$$\frac{1}{L'} = \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{0.5} + \frac{1}{0.5}$$

$$\frac{1}{L'} = 2+2 = 4 \Rightarrow L' = \frac{1}{4} = 0.25\,H$$

Now, L_1 & L' form a series combination. Their equivalent inductance is

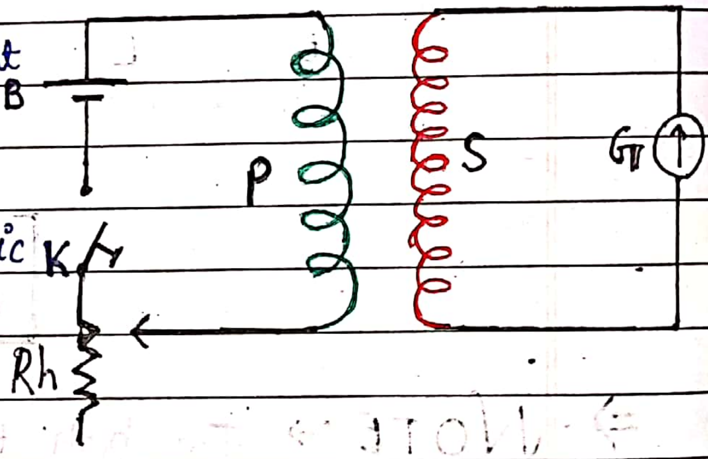


$$L = L_1 + L' = 0.75 + 0.25$$

$$\boxed{L = 1H}$$ Ans.

MUTUAL INDUCTION

'The phenomenon of production of induced emf in a coil due to change in current or magnetic flux linked with its neighbouring coil is called Mutual Induction.'



Ref to fig. When a current flows through coil P, it produces a magnetic field which produces a magnetic flux through coil S. If the current in the coil P is varied, the magnetic flux linked with the coil S changes which induces an emf and hence a current in it.

The coil P is called the PRIMARY COIL and S, the SECONDARY COIL.

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COEFFICIENT OF MUTUAL INDUCTION

At any instant,

Magnetic flux linked with the secondary coil $\propto$ current in the primary coil

$$\phi \propto I$$

$$\therefore \quad \boxed{\phi = MI} \quad \text{—— (1)}$$

M is called **COEFFICIENT OF MUTUAL INDUCTION** or **MUTUAL INDUCTANCE** of two coils.

from eqn (1) $\quad M = \dfrac{\phi}{I}$,

if $\quad I = 1 \quad$ then $\quad M = \phi$

$\therefore$ Mutual Inductance of two coils is numerically equal to the magnetic flux linked with a coil when unit current is passed through its neighbouring coil.

If e is the induced emf in the secondary coil due to the change in flux or current in the primary coil,

then $\quad e = -\dfrac{d\phi}{dt} = -\dfrac{d(MI)}{dt} \quad [\because \phi = MI]$

$$\boxed{e = -M\frac{dI}{dt}} \quad \text{or} \quad \boxed{e = M\frac{dI}{dt}} \text{ (in magnitude)}$$

Now,

$$M = \frac{e}{dI/dt}$$

If $\dfrac{dI}{dt} = 1\ A/s$, then $\boxed{M = e}$

$\therefore$ coefficient of Mutual Induction or mutual inductance of two coils is numerically equal to the emf induced in one coil when rate of change of current in the neighbouring coil is unity.

SI Unit of M :→ as $M = \dfrac{e}{dI/dt}$

$\therefore$ SI unit of $M = \dfrac{V}{A s^{-1}} = V s A^{-1} = \text{Henry (H)}$

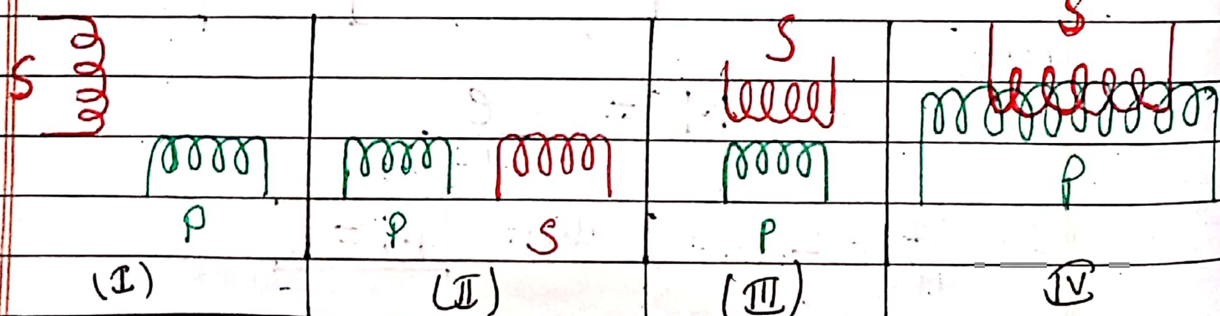
$$1 \text{ Henry} = \frac{1 V}{1 A s^{-1}}$$

⇒ **1 Henry** :→ The coefficient of Mutual Induction of two coils is said to be one Henry if the current changing at the rate of 1 A/s in one coil, induces an emf of 1 V in its neighbouring coil.

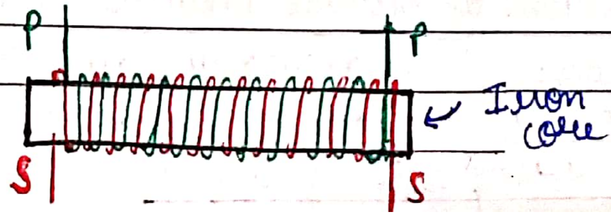
FACTORS AFFECTING MUTUAL INDUCTION B/W TWO COILS :→

Mutual Induction b/w the two coils depends upon:→

(1) Shape and size of the coils.
(2) Separation b/w them.
(3) Permeability of the material of the core (if any) on which the two coils are wound.
(4) The manner in which the two coils are oriented relative to each other.



(V)



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Out of the first four fig.; Mutual Induction is maximum in fig (4). and minimum in fig (I). Mutual Induction in fig(4) can be further increased when the two coils are wound over the soft iron core (fig V).

MUTUAL INDUCTION OF TWO LONG SOLENOIDS

Let the two coils S_1 and S_2 are wound coaxially as shown in fig.

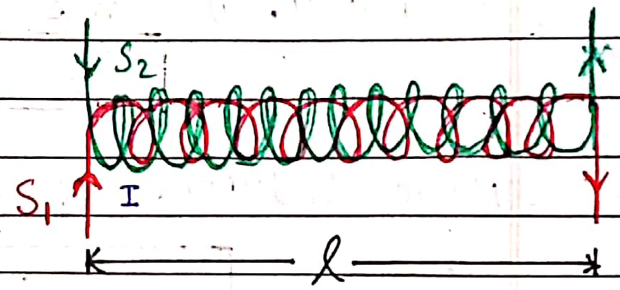
Let,

$l \Rightarrow$ length of each coil.

$n_1 \Rightarrow$ no. of turns per unit length in S_1.

$n_2 \Rightarrow$ no. of turns per unit length in S_2.

A ⇒ common area of two coils.



Let current I flows through S_1

Then magnetic field within S_1

$$B_1 = \mu_0 n_1 I \quad \text{——— (1)}$$

Magnetic flux linked with each turn of S_2

$$\phi_{21}' = B_1 A = \mu_0 n_1 I A \quad \text{--- ( )}$$

Total magnetic flux linked with S_2

$$\phi_{21} = \mu_0 n_1 I A \times \text{total no. of turns in } S_2$$

$$\phi_{21} = \mu_0 n_1 I A \times n_2 l$$

$$\phi_{21} = \mu_0 n_1 n_2 l I A \quad \text{——— (2)}$$

as $\phi = M I$

$\therefore \quad \phi_{21} = M_{21} I$ —— (3)

from eqⁿ (2) and (3), we get

$$M_{21} I = \mu_0 n_1 n_2 l I A$$

$$\boxed{M_{21} = \mu_0 n_1 n_2 l A} \quad \text{—— (4)}$$

Mutual inductance of coil 2 with respect to coil 1.

Similarly, it can be proved

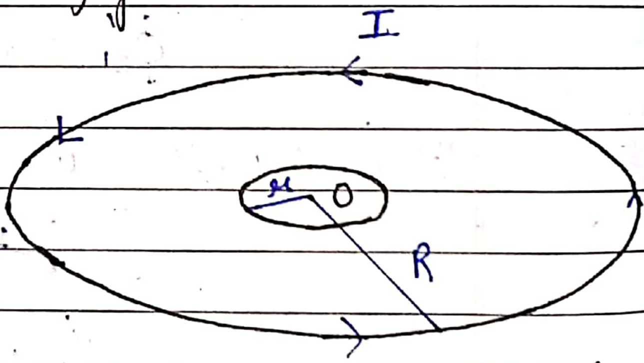
$$\boxed{M_{12} = \mu_0 n_1 n_2 l A} \quad \text{—— (5)}$$

Mutual inductance of coil 1 w.r.t. to coil 2.

i.e., $$\boxed{M_{12} = M_{21} = \mu_0 n_1 n_2 l A} \quad \text{—— (6)}$$

NOTE :-> Thus, mutual induction of two coils is the property of their combination. It does not matter which of them functions as the primary or the secondary coil. This fact is known as RECIPROCITY THEOREM.

Q => Find the mutual inductance of two cocentric circular coils as shown in fig. below.



Solⁿ => Let the current 'I' flows through the coil of radius 'R'.

$\therefore$ Magnetic field due to it at centre O,

$$B = \frac{\mu_0}{4\pi} \frac{2\pi I}{R} \quad \text{—— (1)}$$

$\boxed{r << R}$

So that M.F. due to bigger coil is constant at centre.

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Magnetic flux through coil of radius 'r'

$$\phi = BA = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}(\pi r^2)$$

$$\phi = \frac{\mu_0 \pi I r^2}{2R} \quad \text{——} \ (2)$$

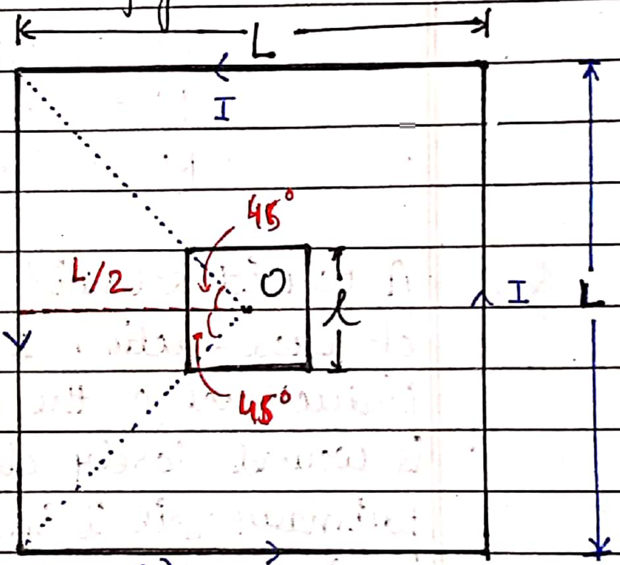
as $\phi = MI$ —— (3)

$\therefore$ from eq^n (2) & (3)

$$M\cancel{I} = \frac{\mu_0 \pi \cancel{I} r^2}{2R}$$

$$\boxed{M = \frac{\mu_0 \pi r^2}{2R}}$$

Q⇒ Find the mutual inductance of two cocentric squared shape coils as shown in fig. ($L >> l$)



Sol^n⇒ Let a current I flows through the square coil of length 'L'

$\therefore$ Magnetic field due to it at centre O,

B = 4 x Magnetic field due to each side

$$= 4 \times \frac{\mu_0}{4\pi}\frac{I}{L/2}(\sin 45^\circ + \sin 45^\circ)$$

$$B = \frac{2\mu_0 I}{\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{2\mu_0 I}{\pi L}\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$

Magnetic flux linked with small square coil of side 'l'

$$\phi = BA = Bl^2 = \frac{2\sqrt{2}\mu_0 I}{\pi L} l^2 \quad \text{——} \ (1)$$

as $\phi = MI$ —— (2)

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$\therefore$ from ① & ②, $M\cancel{I} = \frac{2\sqrt{2}\,\mu_0 \cancel{I} l^2}{\pi L}$

$$\therefore \boxed{M = \frac{2\sqrt{2}\,\mu_0 l^2}{L}}$$

Q⇒ A solenoid coil has 50 turns per cm along its length and a cross-sectional area of 4 cm². 200 turns of another wire is wound around the 1st solenoid coaxially. If the coils are electrically insulated from each other. Calculate the mutual inductance b/w the two coils.

Soln ⇒ $n_1 = 50$ turns/cm $= 5000$ turn/m

$N_2 = n_2 l = 200$, $A = 4 \times 10^{-4}\ m^2$

as $M = \mu_0 n_1 (n_2 l) A$

$\therefore M = 4\pi \times 10^{-7} \times 5000 \times 200 \times 4 \times 10^{-4}$

$M = 160\pi \times 10^{-6}$

$M = 5.02 \times 10^{-4}$ H Ans.

Q⇒ A toroid solenoid has an average radius of 15 cm. Area of cross-section 12 cm² and 1200 turns. Obtain the self inductance of the toroid. If the second coil of 300 turns is wound closely above the 1st and the current in the primary coil is increased from 0 to 2A in 0.05 sec. Obtain the induced emf in second coil.

Soln ⇒ (a) n (no. of turns per unit length of toroidal solenoid) $= \frac{N}{2\pi r} = \frac{1200}{2\pi r}$

Magnetic field set up inside solenoid

$$B = \mu_0 n I = \frac{\mu_0 N}{2\pi r} I$$

$\therefore$ Total flux linked with N turns is

$$\phi_{tot} = NBA$$

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$\therefore \quad LI = N\left(\frac{\mu_0 N}{2\pi r} I\right) A \qquad [\text{as } \phi = LI]$

$$L = \frac{\mu_0 N^2 A}{2\pi r}$$

$$L = \frac{4\pi \times 10^{-7} \times (1200)^2 \times 12 \times 10^{-4}}{2\pi \times (15) \times 10^{-2}}$$

$$L = 2304 \times 10^{-6} \text{ H}$$

$$L = 2.304 \times 10^{-3} \text{ H} = 2.304 \text{ mH}$$

(b) Here, $N_1 = 1200$, $N_2 = 300$, $dt = 0.05\,s$,

$dI = 2 - 0 = 2A$.

$\therefore$ emf induced in the 2^{nd} coil is

$$e = M\frac{dI}{dt} = \frac{\mu_0 N_1 N_2 A}{l}\frac{dI}{dt}$$

$$e = \frac{(4\pi \times 10^{-7}) \times (1200) \times (300) \times (12 \times 10^{-4})}{2\pi \times (0.15)} \times \frac{2}{0.05}$$

$\boxed{e = 0.023\,V} \qquad [\because l = 2\pi r]$

Q⇒ Calculate the mutual inductance b/w two coils if a current of 10 A in the primary coil changes the flux by 500 Wb per turn in the secondary coil of 200 turns. Also determine the induced emf across the ends of the secondary coil in 0.5 s.

Solⁿ⇒ Total flux linked through the secondary coil is

ϕ_{tot} = total turns × flux through each turn

$\phi_{tot} = 200 \times 500$

as $\phi_{tot} = MI$

$\therefore \quad MI = 200 \times 500$

$$M = \frac{200 \times 500}{10} = 10^4 \text{ H}$$

Also, induced emf across secondary coil

$$e = N\frac{d\phi}{dt} = 200 \times \frac{500}{0.5} = 2 \times 10^4 \text{ V}$$

Alternating Current Generator [A.C. Dynamo]

'It is a device which converts mechanical energy into electrical energy.'

PRINCIPLE :→ Its working is based on the principle of Electromagnetic Induction.

'When a closed coil is rotated in a uniform magnetic field with its axis ⊥ to the magnetic field, the magnetic flux linked with the coil changes & an induced emf & hence a current is set up.'

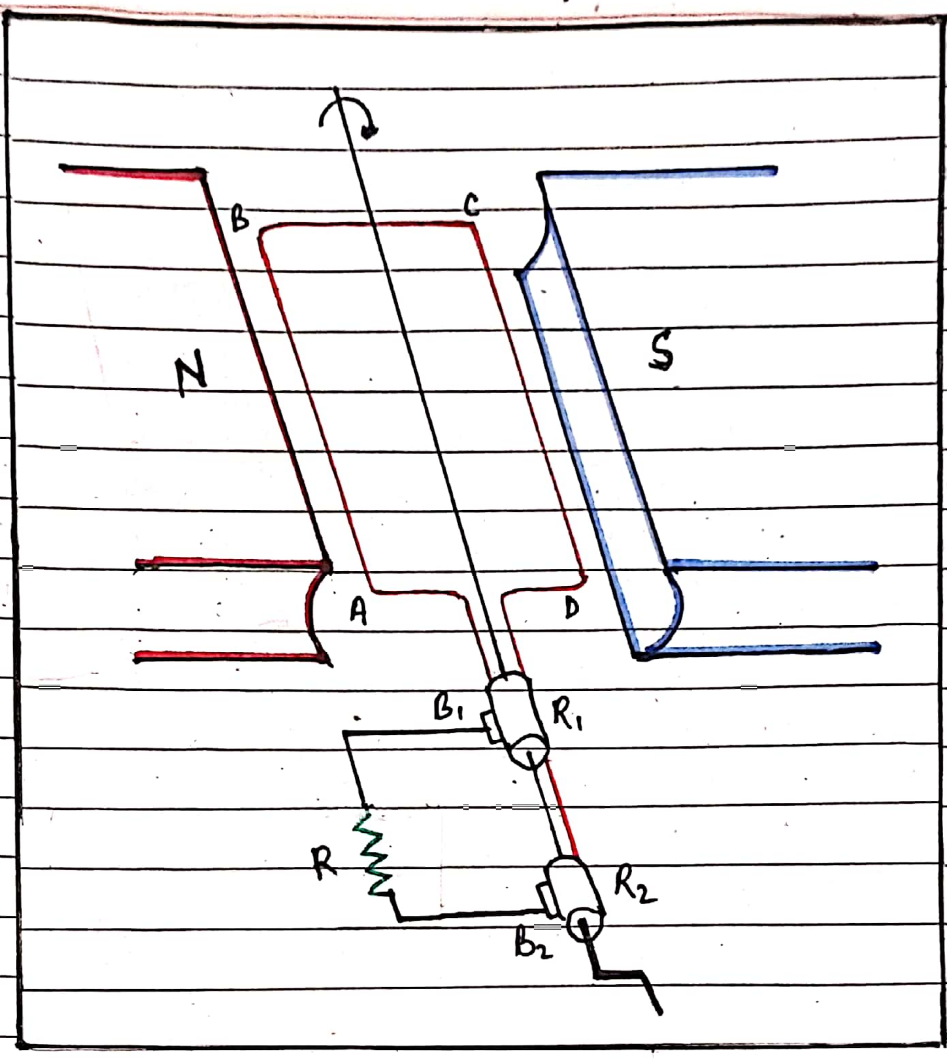
CONSTRUCTION :→

(1) **Armature** ⇒ A rectangular coil ABCD consisting of large no. of turns of copper wire wound over a soft iron core is called armature.

(2) **Field Magnet** ⇒ It is usually a strong permanent magnet having concave poles. The armature has to be rotated b/w the poles of magnet keeping its axis ⊥ to the magnetic field.

(3) **Slip Rings** ⇒ The two ends of the armature coil are connected to two coaxial brass rings R_1 and R_2 separately called slip rings. They rotate with armature.

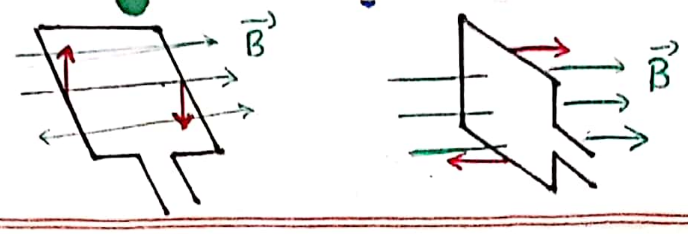
(4) **Brushes** ⇒ These are flexible metallic pieces (B_1 and B_2) used to pass the current from armature to the external load (R). As the slip rings rotate, the brushes provide movable contact by keeping themselves pressed against the rings.



WORKING :→ The working of A.C. Generator is illustrated with the help of five different orientations of the armature ABCD at the time intervals $t=0$, $t=T/4$, $t=T/2$, $t=3T/4$, $t=T$ respectively. (ref to fig. on next page).

(1) **Direction of flow of current** :→

Let at $t=0$ armature ABCD is vertical with AB up & CD down. Between $t=0$ and $t=T/2$ AB moves down and CD moves up. Using Fleming's Right Hand rule, the direction of induced current in arm AB and CD will be from B to A and D to C respectively. For next half rotation (b/w $t=T/2$ to $t=T$), arm CD moves down and AB moves up and hence the direction of induced current in two arms will get reversed.

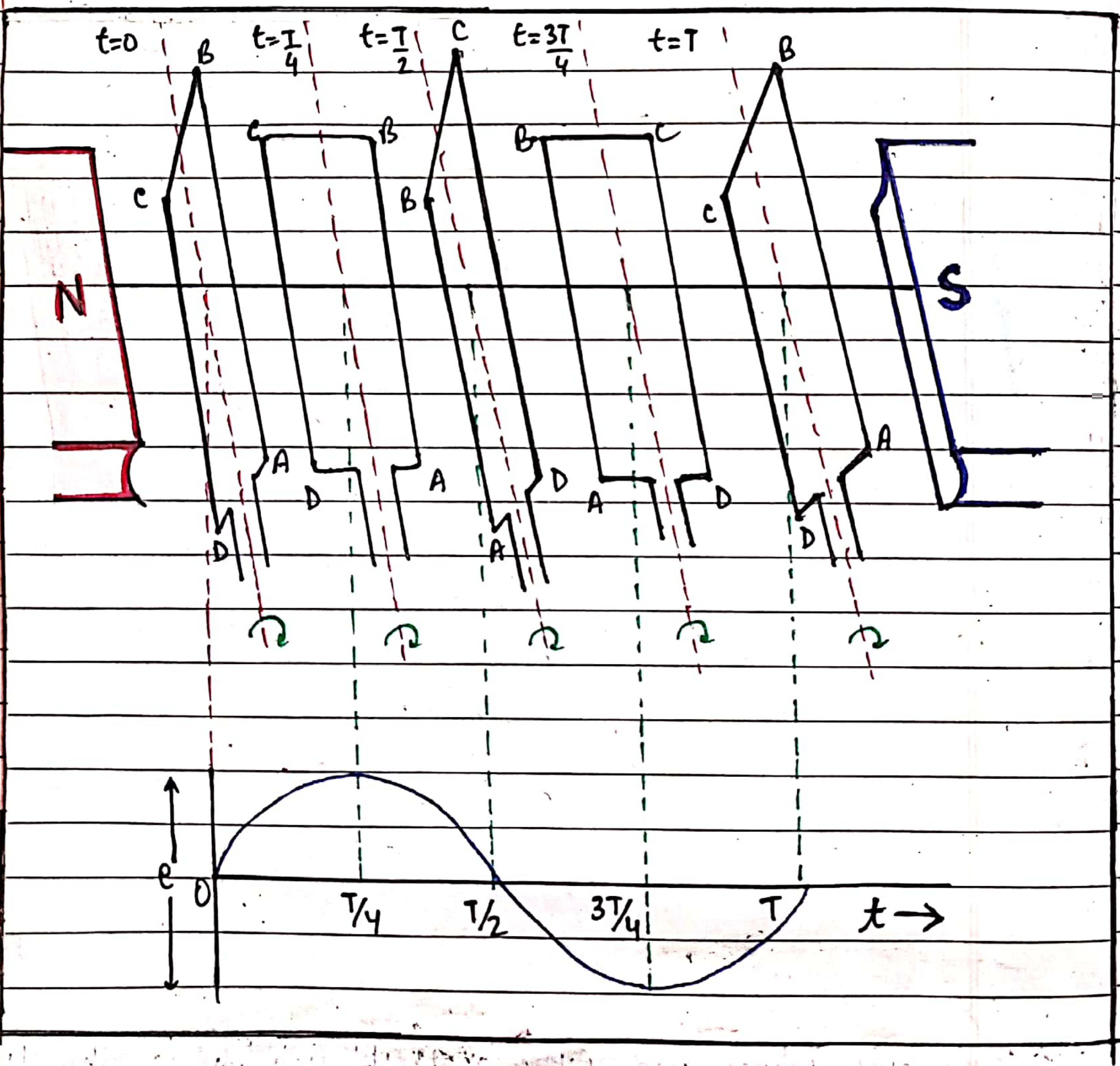


(2) Magnitude of Induced emf :-

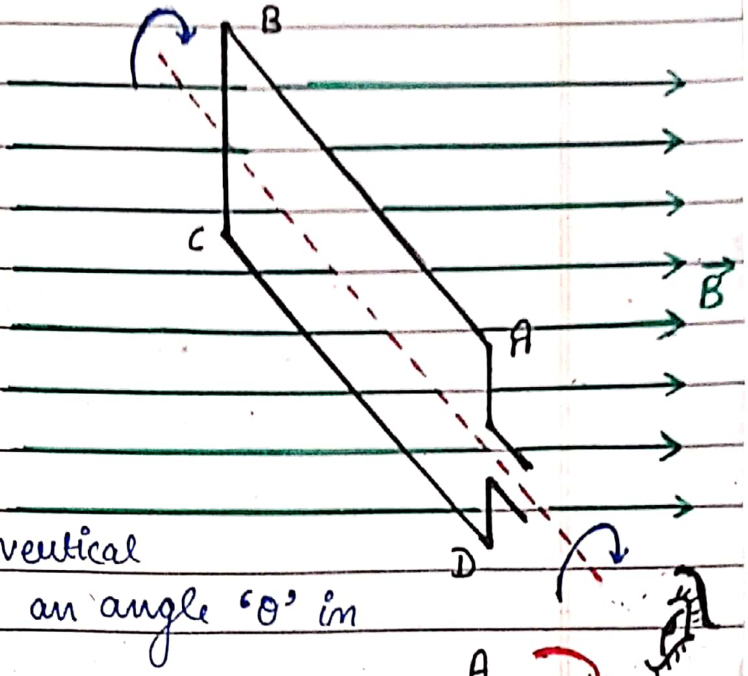
Whenever the armature is vertical, the arms AB & CD momentarily moves parallel to the field i.e., $\frac{d\phi}{dt}$ (rate of change of flux) = 0 and hence instantaneous emf, $e = 0$ $\left[\text{as } |e| = \frac{d\phi}{dt}\right]$. Thus, at $t = 0$, $t = T/2$ and $t = T$, instantaneous emf, $e = 0$.

Whenever the armature is horizontal, its arms AB & CD momentarily moves perpendicular to the magnetic field.

$\therefore \frac{d\phi}{dt}$ = maximum & hence $|e|$ = max. Thus, at $t = T/4$ and $t = 3T/4$, $|e|$ = max.



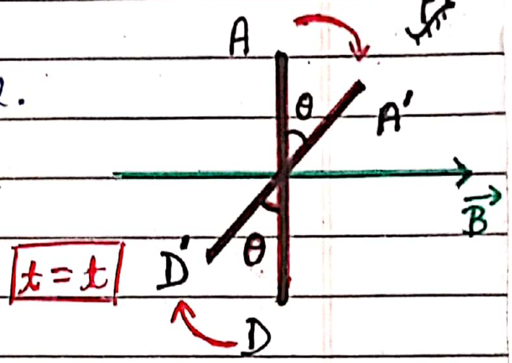
Expression for Instantaneous emf Produced



Let the coil is rotating with uniform angular speed 'ω' in a uniform magnetic field B as shown.

Let at $t = 0$, coil is vertical and it is turned by an angle 'θ' in time $t = t$.

Let N = No. of turns in the coil.



As $\omega = \dfrac{\theta}{t}$ $\quad \therefore \theta = \omega t$ ——— (1)

Instantaneous magnetic flux linked with the coil

$$\phi = NBA\cos\theta$$

$$\phi = NBA\cos\omega t \quad \text{——— (2)}$$

as $$e = \frac{-d\phi}{dt}$$

$$e = -\frac{d(NBA\cos\omega t)}{dt}$$

$$e = -NBA\frac{d(\cos\omega t)}{dt}$$

$$e = -NBA(-\sin\omega t)\cdot\omega$$

$$\boxed{e = NBA\omega\sin\omega t} \quad \text{——— (3)}$$

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for max. emf (say e_0),
$\sin \omega t$ should be maximum i.e., $\sin \omega t = 1$

$$\therefore \quad \boxed{e_0 = NBA} \quad — (4)$$

$\therefore$ from eqn (3) & (4)

$$\boxed{e = e_0 \sin \omega t} \quad — (5)$$

Q⇒ A 100 turns coil of area $0.1\,m^2$ rotates at a half revolution per second in a uniform magnetic field of $0.01\,T$ which is perpendicular to its axis of rotation. Find the maximum emf induced.

Soln⇒ Here, $N = 100$ $\quad A = 0.1\,m^2$ $\quad \nu = \frac{1}{2}$ rev/sec $\quad B = 0.01\,T$

as $\quad e_0 = NBA\omega$

$e_0 = (100) \times (0.01) \times (0.1) \times 2\pi\nu \quad [\because \omega = 2\pi\nu]$

$e_0 = 0.1 \times 2 \times 3.14 \times \frac{1}{2}$

$\boxed{e_0 = 0.314\,V}$

Q⇒ A rectangular coil of dimensions $0.1\,m \times 0.5\,m$ consisting of 2000 turns rotates about an axis parallel to its longer side making 2100 revolutions per minute in a field of $0.1\,T$. What is the maximum emf induced in the coil? Also find the instantaneous emf, when the coil is at 30° to the field.

Soln⇒ Here, $A = 0.1\,m \times 0.5\,m = 5 \times 10^{-2}\,m^2$, $N = 2000$

$\nu = 2100$ rpm $= \frac{2100}{60}$ rps $= 35$ rps, $B = 0.1\,T$

$e_0 = NBA\omega$

$e_0 = 2000 \times 0.1 \times 5 \times 10^{-2} \times 2\pi\nu$

$e_0 = 20\pi \times 35 = 700\pi$

$e_0 = 700 \times \frac{22}{7} = 2200\,V$

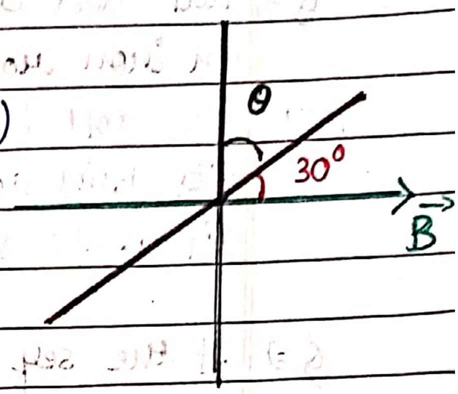
as $e = e_0 \sin \omega t$

$e = e_0 \sin \theta$

$e = 2200 \sin (90^\circ - 30^\circ)$

$e = 2200 \times \frac{\sqrt{3}}{2}$

$$\boxed{e = 1100\sqrt{3}\ V}$$



⇒ CONCEPTUAL QUESTIONS :⇒

Q⇒ Why is spark produced in the switch of a fan, when it is switched off?

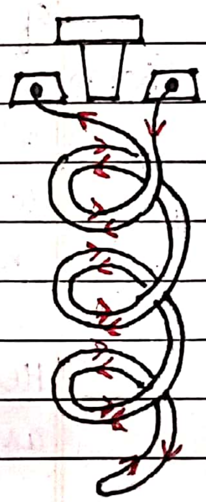
Ans⇒ The break of circuit is very sudden and as a result there is sudden change in the magnetic flux. Due to this a large induced emf is set up across the gap in the switch (self-induction) due to which sparking occurs.

Q⇒ A coil is wound on an iron core and looped back on itself, so that the core has two sets of closely wound wires in series, carrying currents in the opposite senses. What do you expect about its self inductance? Will it be large or small?

Ans⇒ The self Inductance will be SMALL. The inductive effects in the two wires will be in opposite directions and hence cancel each other. The net self-inductance of the coil is minimum.

Such a winding of coils is called

NON-IDUCTIVE WINDING.



Non-Inductive winding of resistance coil

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Q⇒ How does the self-inductance of a coil change when an iron rod is introduced in it?

Ans⇒ The soft iron has a large relative permeability (μ_r). Its presence increases the magnetic flux μ_r times. The self inductance also increases by the same ratio.

Q⇒ If the self inductance of an air core inductor increases from 0.01 mH to 10 mH on introducing an iron core into it, what is the relative permeability of the core used?

Solⁿ⇒ With air core,

$$L_0 = \frac{\mu_0 N^2 A}{l}$$ [μ_0 ⇒ permeability of air]

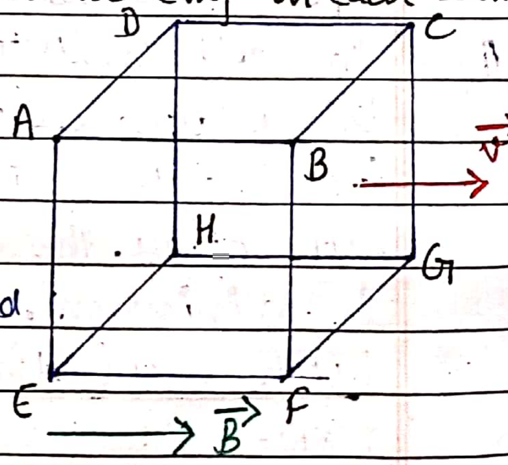
With iron core, $$L = \frac{\mu N^2 A}{l}$$ [μ ⇒ permeability of iron].

Now, $$\frac{L}{L_0} = \frac{\mu}{\mu_0} \Rightarrow \frac{L}{L_0} = \mu_r$$ [as $\mu_r = \frac{\mu}{\mu_0}$]

Relative permeability

$$\therefore \mu_r = \frac{10}{0.01} = 1000.$$

Q⇒ 12 wires of equal lengths are connected in the form of a skeleton-cube which is moving with a velocity $\vec{v}$ in the direction of a magnetic field $\vec{B}$. Find the emf in each arm of the cube.



Solⁿ⇒ As the direction of velocity $\vec{v}$ of any arm of the cube is ∥el to the direction of field $\vec{B}$, so no magnetic Lorentz force is exerted on the free e⁻s of any arm. Hence no emf is induced in any arm.

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Q) A circular brass loop of radius 'a' and resistance 'R' is placed with its plane $\perp$ to a magnetic field which varies with time as $B = B_0 \sin\omega t$. Obtain the expression for the induced current in the loop.

Sol^n) Here, $A = \pi a^2$, $\theta = 0°$. So flux linked with the loop is,

$$\phi = BA\cos 0° = BA$$

$$\phi = B_0 \sin\omega t \,(\pi a^2)$$

$\therefore$ induced emf, $|e| = \dfrac{d\phi}{dt} = \dfrac{d}{dt}(B_0 \sin\omega t \cdot \pi a^2)$

$$|e| = \pi a^2 B_0 \frac{d}{dt}(\sin\omega t)$$

$$|e| = \pi a^2 B_0 (\cos\omega t) \cdot \frac{d}{dt}(\omega t)$$

$$|e| = \pi a^2 B_0 \omega \cos\omega t$$

Induced current,

$$I = \frac{e}{R} = \frac{\pi a^2 B_0 \omega \cos\omega t}{R}$$

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